2050 B HW III

1. Let $\emptyset \neq A \subseteq B \subseteq \mathbb{R}$ and B bounded below.
Show that $\inf A \geq \inf B$
(and establish the corresponding result for sup).

2 Let $\emptyset \neq A \subseteq \mathbb{R}$ and A bounded below with $\alpha = \inf A$. Prove/disprove for each of the following assertions (i.e. prove each correct assertion and provide a counter-example for each incorrect assertion):

(i) If $\alpha \leq x$ then $\exists\, a \in A$ s.t. $a \leq x$;

(ii) If $\alpha < x$ then $\exists\, a \in A$ s.t. $a < x$;

(iii) If $\alpha < x$ then $a \leq x$ for all $a \in A$;

(iv) If $y \leq \alpha$ then $y \leq a$ for all $a \in A$;

(v) If $y \leq \alpha$ then $a \leq a$ for some $a \in A$.

3 Let $z < \sup A$. Show that $z < a$ for some $a \in A$!

4. Show Th1 and Th2 (namely, charaterization theorem for intervals, and nested intervals theorem: state and prove them!)